LEAGUES

MAML

STATE INVITATIONAL MATH LEAGUE COMPETITION

April 1, 2016

Shrewsbury High School

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2016

Round 1 **Arithmetic and Number Theory**

1. ______________________

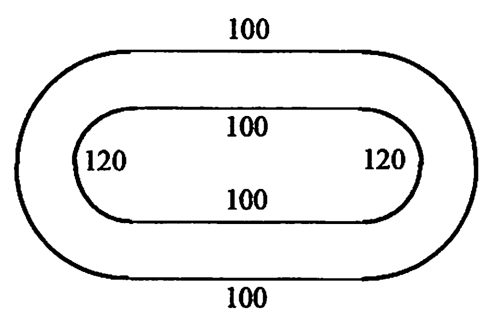
2. ______________________

3. ______________________

1. Marty buys a cubic yard of black dirt. He wants to cover his grass with a layer of dirt $\frac{1}{4}$ inch deep. Compute the number of square feet of grass that the dirt can cover.

2. Compute the number of ordered pairs (A,B) with $A,B \in \{1,2,3,4,5,6,7,8,9\}$ such that $\left(0.\overline{A}+0.\overline{B}\right)^{-1}=\left(0.\overline{AB}+0.\overline{BA}\right)^{-1}$.

3. Let $\pi = 3.1$. A track consists of two 100-yard straightaways and two 120-yard semicircles. A lane is a yard wide. Running on the very inside of the first lane a runner covers a mile in exactly 4 laps. If the runner runs on the very inside of the 5th lane, how many laps constitute a 1-mile run? Round your answer to the nearest tenth.



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Round 2 **Algebra 1**

1. ________________

2. ________________

3. ________________

1. Find all solutions to $\left(x^2-x-6\right)^2-\left(x^2-9\right)^2=0$.

2. Compute the smallest solution to $4\sqrt{|x-1|}=-\frac{x}{2}+4$.

3. Given that the equation $x^2+Kx+J=0$ has real roots r_1 and r_2, where $r_2=3\cdot r_1$.
If $JK=-96$, compute $\frac{K}{J}$.

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Round 3 – Geometry

1. ______________________

2. ______________________

3. ______________________

1. In isosceles trapezoid $ABCD$, the sum of the lengths of the bases $\overline{AB}$ and $\overline{CD}$ is 28 cm. The length of the longer base is $\frac{9}{5}$ the length of the shorter base. The area of the trapezoid is 84 sq. cm. Determine the exact number of centimeters in each of the non-parallel sides.

2. In some order, the lengths of the sides of ΔABC are 8, 10, and 12. Let K lie on $\overline{BC}$ such that $\overline{AK}$ bisects $\angle BAC$. If $\frac{AB+BK}{AC+CK}=\frac{2}{3}$, compute $AB+CK$.

3. The length of a side of a regular 12-sided polygon is 2. Compute the apothem of the polygon.

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Round 4 – Algebra 2

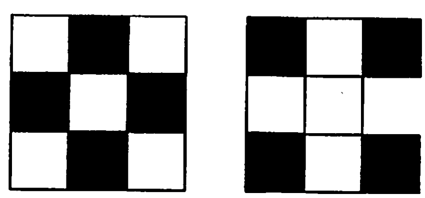
1. ____________________

2. ____________________

3. ____________________

1. Let $\left[x\right]$ denote the greatest integer less than or equal to x. Compute the value of $2016^{\frac{1}{\ln 2016}}$.

2. A 3 by 3 square is divided into nine 1 by 1 squares and 4 of the squares are painted black as shown in the diagram at the left. What is the probability that, if the 1 by 1 squares are cut out and then reassembled at random, the 3 by 3 square at the right will be produced?



3. An arithmetic progression and a geometric progression have the same first term, $a_1 = g_1 = 2$. The second term of the geometric progression is 9 less than the second term of the arithmetic progression, and the third term of both progressions are equal.
Compute <u>all</u> possible ordered pairs (a_4, g_4). Answers must be in proper ordered pair notation.

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Round 5 – Analytic Geometry

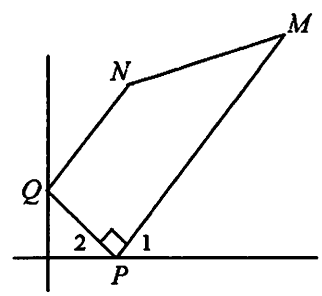
1. ______________________

2. ____(________,________)__

3. ____(________,________)__

1. Line $\overleftrightarrow{AB}$ has equation $y = 2x + 3$ with point A on the y-axis and a horizontal change of +2 to B. Line $\overleftrightarrow{BC}$ is perpendicular to line $\overleftrightarrow{AB}$ with C on the x-axis. Determine the number of square units in the area of quadrilateral $ABCO$ given that O is the origin.

2. In trapezoid $NMPQ$ of area 20, Q lies on the y-axis, P is on the x-axis, $\overline{MP} \perp \overline{QP}$, and $\angle 1 \cong \angle 2$. If $M = (9,7)$, find the coordinates of N.



3. Given the equation of a parabola, $(y + 1)^2 = -8(x + 3)$. From $P(-11, 7)$ on the parabola, a line passes through the focus F, and then intersects the parabola at point Q. Find the coordinates of point Q.

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Round 6 – Trig and Complex Numbers

1. ____________________

2. ____________________

3. ____________________

1. Given $m\angle A = 50°$, $m\angle B = 55°$, $m\angle C = 40°$, $m\angle D = 25°$, compute the value of $\sin(A + B) - \sin(D - C)$.

2. Let P be the intersection of the diagonals of rectangle $ABCD$. If $ABCD$ is a golden rectangle where side $\overline{AB}$ is the length and side $\overline{BC}$ is the width, compute $\tan \angle BPC$. (Note: a golden rectangle is one in which $\frac{\text{length}}{\text{width}} = \frac{\text{length + width}}{\text{length}}$. The ratio length : width is called the golden ratio)

3. Let O be the origin in the complex plane, let $P = a + bi$, and $Q = c + di$. The arguments of both P and Q are positive acute angles and each has a modulus of 2. Let $M = P^2$ and $N = Q^2$. The ratio of the area of ΔOMN to the area of ΔOPQ is 3. Compute the cosine of the difference of the arguments of P and Q.

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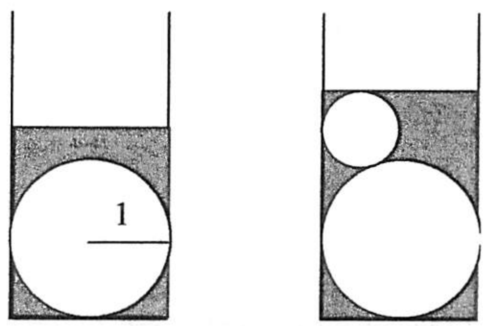
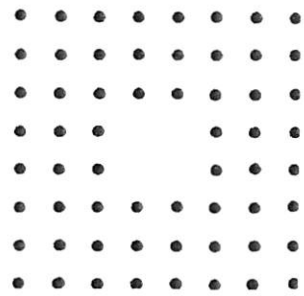
Team Round

Place answers on Team Round Answer Sheet

1. Water is poured into a cylinder of radius 1 and when a metal sphere of diameter 1 is placed in the cylinder the water completely fills the space below the sphere. Additional water is then poured into the cylinder until the water level reaches a height of 1/2 inch above the sphere. as shown on the left. A smaller sphere of radius r is then placed inside the cylinder as shown and it raises the water level up to the top of the smaller sphere. The radius r of the smaller sphere is a solution to the equation $Ar^3 + Br + C\sqrt{r} + 9 = 0$. Compute the ordered triple (A, B, C).



2. A group of N people forms a hollow square with sides 3 deep. As an example, an analogous hollow square is shown below. If 25 people are added to the group, a solid square can be formed, each side of which is 22 more than the square root of the number of people on each <u>outer side</u> of the original hollow square. Compute N.



3. Let $a_1 = k$, $a_2 = m$, and for $n > 2$, $a_n = a_{n-1} + a_{n-2}$. If m and k are positive integers and $a_7 = 2016$, determine the number of possible values of k such that $k > m$.

4. Let $\left[x\right]$ denote the greatest integer less than or equal to x. Compute all real solutions to $x^2 - 2\left[x\right] - 3 = 0$.

5. For $0 \le x \le 1$, compute the least value taken on by $\dfrac{1}{|x-0.3|} + \dfrac{1}{|x-0.8|}$.

6. Let $T = \{2,3,4,5,\ldots,10\}$. Compute the number of sets of three distinct numbers from T that could be used as lengths of the sides of non-degenerate triangles.

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STATE PLAYOFFS – 2016 – ANSWER SHEET

Round 1

1. 1296
2. 81
3. 3.8

Round 2

1. 3 or $-\frac{5}{2}$
2. -48
3. $-\frac{2}{3}$

Round 3

1. $2\sqrt{13}$
2. 14
3. $2+\sqrt{3}$

Round 4

1. 2
2. $\frac{1}{126}$
3. $(47,128)$
   $(11,-16)$

Round 5

1 59

2. $(3,5)$
3. $\left(-\frac{7}{2},-3\right)$

Round 6

1. $\frac{\sqrt{6}}{2}$
2. 2
3. $\frac{3}{8}$

Team

1. $(8,-6,-12)$
2. 936
3. 31
4. $-1, \sqrt{7}, 3$
5. $\frac{55}{12}$
6. 49

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Solutions: State Meet 2016

Round 1 Arithmetic and Number Theory:

1. Let x = the number of feet on a side, so $12x$ equals the number of inches on a side. The volume in inches of a cubic yard is 36^3 so $\frac{1}{4}(12x)^2 = 36^3 \to x^2 = 36^2 = \boxed{1296 \text{ square feet}}$

2. Since $.\overline{A} = \frac{A}{9}$, the left side equals $\left(\frac{A}{9}+\frac{B}{9}\right)^{-1} = \frac{9}{A+B}$. Since $.\overline{AB} = \frac{10A+B}{99}$, the right side equals $\left(\frac{10A+B}{99}+\frac{10B+A}{99}\right)^{-1} = \left(\frac{11A+11B}{99}\right)^{-1} = \left(\frac{A+B}{9}\right)^{-1} = \frac{9}{A+B}$. Thus, the two expressions are equivalent, so all possible ordered pairs work. There are 9 choices for A and 9 choices for B making a total of 9 times 9 = $\boxed{81}$ solutions.

3. Think of the two semicircles as one circle with a radius of r. Then the radius of the inside of the 5th lane is $r+4$ and the difference in circumference is $2\pi(r+4)-2\pi r = 8\pi$. Each lap in the 5th lane is $440+8\pi = 464.8$ yards. $\frac{1760}{464.8} = 3.7865$ Answer: $\boxed{3.8}$.

Round 2 Algebra I:

1. $\left(x^2-x-6\right)^2-\left(x^2-9\right)^2=0$ gives

$\left((x-3)(x+2)\right)^2-\left((x-3)(x+3)\right)^2=0 \to (x-3)^2\left((x+2)^2-(x+3)^2\right)=0 \to (x-3)^2(-2x-5)=0$. Thus, $\boxed{x=3 \text{ or } -\frac{5}{2}}$.

2. Start by guessing that there is a solution less than 1, making $|x-1| = 1-x$. Then $\left(4\sqrt{1-x}\right)^2 = \left(4-\frac{x}{2}\right)^2$ gives $16-16x = \frac{x^2}{4}-4x+16 \to x^2+48x=0$. Thus, $x=0$ or -48. The smallest is $\boxed{x=-48}$.

3. $\begin{cases} r_1 + r_2 = -K \\ r_1 r_2 = J \\ r_2 = 3r_1 \end{cases} \Rightarrow \begin{cases} K = -4r_1 \\ J = 3r_1^2 \end{cases} \Rightarrow JK = -12r_1^3 = -96 \Rightarrow r_1 = 2,\ r_2 = 6$

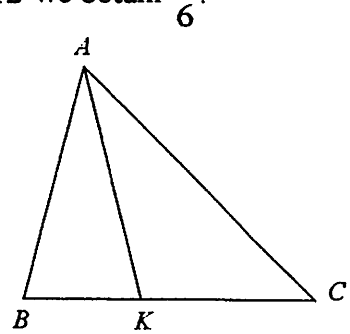
Thus, $\frac{K}{J} = \frac{-8}{12} = \underline{-\frac{2}{3}}$

Round 3 **<u>Geometry</u>:**

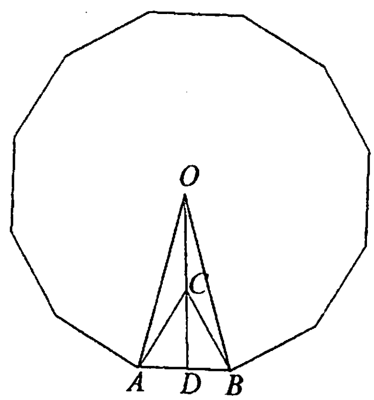
1. $\frac{9}{5}x + x = 28 \rightarrow x = 10$. The bases are 10 and 18. $\frac{1}{2}(10 + 18)h = 84 \rightarrow h = 6$. Let $\overline{AE} \perp \overline{DC}$.

In Rt. ΔAED $AD^2 = 6^2 + 4^2 \rightarrow AD = 2\sqrt{13}$.

2 Consider the diagram below. By the Triangle Angle Bisector Theorem, $\frac{AB}{AC} = \frac{BK}{CK}$. If $AB = 8$ and $AC = 12$, then $BK = 8x$ and $CK = 12x$, giving $\frac{AB + BK}{AC + CK} = \frac{8 + 8x}{12 + 12x} = \frac{8(1+x)}{12(1+x)} = \frac{2}{3}$, showing that $AB = 8$, $AC = 12$, and $BC = 10$ fits the conditions of the problem. Then $BC = 8x + 12x = 20x = 10$ so $x = \frac{1}{2}$, making $CK = 6$. Then $AB + CK = 8 + 6 = \boxed{14}$. Had we chosen $AB = 8$ and $AC = 10$ and pursued the same reasoning, we would have obtained $\frac{4}{5}$ for a ratio, not $\frac{2}{3}$. With $AB = 10$ and $AC = 12$ we obtain $\frac{5}{6}$.



3. Construct an equilateral triangle ABC on side $\overline{AB}$ of the polygon and connect the center of the polygon O with vertex C of the triangle and continue the line until it reaches $\overline{AB}$ at D. Since $OA = OB$ and $CA = CB$, $\overline{OD}$ is the perpendicular bisector of $\overline{AB}$. Since $AD = 1$ then $CD = \sqrt{3}$. ΔACO is isosceles since $m\angle AOB = 30°$ making $m\angle AOC = 15°$ and $m\angle OAC = \frac{1}{2}\cdot 150° - m\angle CAD = 75° - 60° = 15°$. Since $AC = 2$ then $CO = 2$, making the apothem equal to $\boxed{2+\sqrt{3}}$.



Round 4 Algebra II

1. $\left[2016^{\frac{1}{\ln 2016}}\right] = \left[2016^{\log_{2016} e}\right] = [e] = \boxed{2}$.

2. The problem asks for the probability that from the arrangement on the left, the arrangement on the right will appear if the squares are re-assembled at random.

X	A	X
A	X	A
X	A	X

A	X	A
X	X	X
A	X	A

This is analogous to asking what is the probability that if 5 indistinguishable X's and 4 indistinguishable A's are arranged in a row at random, that $AXAXXXAXA$ will occur. There are $\frac{9!}{5!\cdot 4!}=126$ possible distinct arrangements. Of those only 1 will have the A's in the appropriate places, so the answer is $\boxed{\frac{1}{126}}$.

3. If the progressions are $\begin{cases} AP: 2, 2+d, 2+2d, \ldots \\ GP: 2, 2r, 2r^2, 2r^3, \ldots \end{cases}$, then $\begin{cases} a_2 = g_2 - 9 \Rightarrow 2r = (2+d)-9 \\ a_3 = g_3 \Rightarrow (2+2d) = 2r^2 \end{cases}$

$\Rightarrow \begin{cases} d = 2r+7 \\ r^2 = d+1 \end{cases} \Rightarrow r^2 - 2r - 8 = (r-4)(r+2) = 0 \Rightarrow (r,d) = (4,15) \text{ or } (-2,3)$

Therefore, $(a_4, g_4) = \begin{cases} (2+45, 2\cdot 4^3) = \underline{\mathbf{(47,128)}} \\ (2+9, 2\cdot(-2)^3) = \underline{\mathbf{(11,-16)}} \end{cases}$

Round 5 **Analytic Geometry:**

1. $A(0,3)$, $B(2,7)$, Line $\overleftrightarrow{BC}$ has equation $y-7=-\frac{1}{2}(x-2)$. That makes $C(16,0)$. Label $(2,0)$ as E. $OABE$ is a trapezoid of area $\frac{1}{2}(3+7)\cdot 2=10$. ΔEBC has area $\frac{1}{2}(16-2)\cdot 7=49$. Therefore, the area of quadrilateral $OABC$ is $10+49=59$.

2. Let P be $(a,0)$ and drop a perpendicular from M to the x-axis at E. ΔPME is isosceles right. $ME=EP=7$. E is $(9,0)$. $PE=9-a=7\rightarrow a=2$. $PO=OQ=2\rightarrow PQ=2\sqrt{2}$
Let N be (r,s) and construct a perpendicular from N to the y-axis at F. 1
Then $FN=QF=r$ and $QN=r\sqrt{2}$. Since the area of the trapezoid is 20, we have
$\frac{1}{2}\cdot 2\sqrt{2}(r\sqrt{2}+7\sqrt{2})=20\rightarrow 2r+14=20\rightarrow r=3$. $\therefore$ N is $(3,5)$

3. The vertex is $(-3,-1)$ and since $4p=8$, $p=2$. That makes the focus $(-5,-1)$. The slope of $\overline{PF}$ is $\frac{-1-7}{-5+11}=-\frac{4}{3}$. The equation of $\overline{PF}$ is $4x+3y=-23$. Substituting $x=\frac{-3y-23}{4}$ into the equation of the parabola gives $(y+1)^2=-8\left[\frac{-3y-23}{4}+3\right]=-2[-3y-11]=6y+22$. Now $y^2+2y+1=6y+22\rightarrow y^2-4y-21=0\rightarrow(y-7)(y+3)=0$. This gives $y=-3$ and $x=-\frac{7}{2}$.
Therefore $Q\left(-\frac{7}{2},-3\right)$

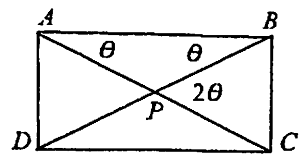
Round 6 **Trigonometry and Complex Numbers:**

1. $\sin(50+55)-\sin(25-40)=\sin(105)-\sin(-15)=\sin(45+60)-\sin(30-45)=$
$\sin(45+60)+\sin(45-30)=\frac{\sqrt{2}}{2}\cdot\frac{1}{2}+\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}}{2}$

2. The golden ratio is $\frac{\sqrt{5}+1}{2}$, so without loss of generality, let $AB=\sqrt{5}+1$ and $BC=2$. Note that ΔPAB is isosceles. Let $m\angle PAB=\theta$, making $\angle PBA=\theta$. By the Exterior Angle Theorem, $m\angle BPC=2\theta$. From $\tan\theta=\frac{BC}{AB}=\frac{2}{\sqrt{5}+1}=\frac{2}{\sqrt{5}+1}\cdot\frac{\sqrt{5}-1}{\sqrt{5}-1}=\frac{\sqrt{5}-1}{2}$ we obtain

$$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}=\frac{2\left(\frac{\sqrt{5}-1}{2}\right)}{1-\left(\frac{\sqrt{5}-1}{2}\right)^2}=\frac{4\left(\sqrt{5}-1\right)}{2\left(\sqrt{5}-1\right)}=2.$$

Since $\angle BPA$ is supplementary to $\angle BPC$, its tangent will be the negative, namely -2. Thus, no matter which $\angle P$ is chosen, $|\tan P|=\boxed{2}$.

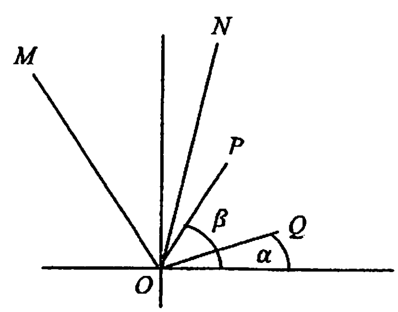


Alternate Solution:

AB = DC = l and AD = BC = w. Then $\frac{l}{w} = \frac{(l+w)}{l} \rightarrow l^2 = lw + w^2 \rightarrow l^3 - w^2 = lw$.

Now $m\angle BPC = 2m\angle BDC = \frac{w}{l}$. $\tan \angle BPC = \tan 2\angle BDC = \frac{2\frac{w}{l}}{1-\frac{w^2}{l^2}} = \frac{2wl}{w^2-l^2} = \frac{2wl}{wl} = 2$.

3. Without loss of generality, let the argument of P be α and the argument of Q be β. Then the area of ΔOPQ is $\frac{1}{2}\cdot 2\cdot 2\sin\left(\alpha-\beta\right) = 2\sin(\alpha-\beta)$. Since squaring a complex number doubles its angle and squares its modulus, the area of $\Delta OMN = \frac{1}{2}\cdot 4\cdot 4\sin\left(2\alpha-2\beta\right) =$ $8\sin\left[2\left(\alpha-\beta\right)\right] = 8\cdot 2\sin(\alpha-\beta)\cos(\alpha-\beta)$. Thus, $\frac{16\sin(\alpha-\beta)\cos(\alpha-\beta)}{2\sin(\alpha-\beta)} = 3$ making $\cos(\alpha-\beta) = \boxed{\frac{3}{8}}$. Note: it doesn't matter which of α or β is larger since the cosine will still be positive since $-90° < \alpha-\beta < 90°$ so the difference lies in the 1st and 4th quadrants.



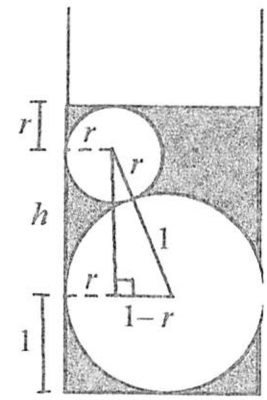
Alternate Solution:

Let $OP = OQ = a$, and $m\angle POQ = \beta$. Then $OM = ON = a^2$ and $m\angle MON = 2\beta$. Now the ratio of the area of ΔMON to the area of ΔPOQ is $\frac{a^4 \sin 2\beta}{a^2 \sin \beta} = 2a^2\cos\beta = 3 \rightarrow \cos\beta = \frac{3}{2a^2} = \frac{3}{8}$, since $a = 2$.

Team:

1. The volume of the water plus the large sphere is $\pi \cdot 1^2 \cdot \left(2+\frac{1}{2}\right) = \frac{5\pi}{2}$.

The volume of the larger sphere is $\frac{4}{3}\pi \cdot 1^3 = \frac{4\pi}{3}$, so the volume of water is $\frac{5\pi}{2} - \frac{4\pi}{3} = \frac{7\pi}{6}$. To find the height of the water after the small sphere has been added we need to find an expression for h. Note that $h^2 = (1+r)^2 - (1-r)^2 = 4r$ so $h = 2\sqrt{r}$. The height of water just touching the top of the small sphere is $1+h+r = 1+2\sqrt{r}+r$. The volume contained by the cylinder can be written as $\pi \cdot 1^2 \cdot \left(r+2\sqrt{r}+1\right)$ or as the sum of the separate parts: $\frac{4\pi}{3} + \frac{7\pi}{6} + \frac{4\pi r^3}{3}$.



Set these two equal, cancel π and set the result equal to 0 to obtain $\frac{4}{3}r^3 - r - 2\sqrt{r} + \frac{3}{2} = 0$. To obtain a constant of 9, multiply by 6 and get $8r^3 - 6r - 12\sqrt{r} + 9 = 0$. So $(A,B,C) = \boxed{(8,-6,-12)}$.

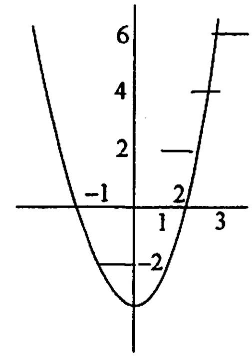
2. Start with an m by m square of people. The outer square has $4m-4$ people since the corners were counted twice in $m+m+m+m$. The first inner square has $4(m-2)-4 = 4(m-3)$ people and the last inner square has $4(m-4)-4 = 4(m-5)$. Thus, $N = 4m-4+4m-12+4m-20 = 12(m-3)$ people. Let t be the number of people on each outer side of the filled in square. Then $12(m-3)+25 = t^2$ and $t-22 = \sqrt{m} \to \sqrt{m}+22 = t$. Thus,

$12m-11 = \left(\sqrt{m}+22\right)^2 \to m-4\sqrt{m}-45 = 0 \to \left(\sqrt{m}-9\right)\left(\sqrt{m}+5\right) = 0$. Then $\sqrt{m} = 9 \to m = 81$,

making $N = 12(81-3) = \boxed{936}$. Note that adding 22 to 936 gives 31^2.

3. $a_7 = 5k+8m \to 5k+8m = 2016 \to m = 252 - \frac{5}{8}a$. Let $a = 8t$, then $m = 252-5t$. We want $8t$ to be greater than $252-5t$ so $8t > 252-5t \to 13t > 252 \to t > 19 \to t \geq 20$. We also want m to be positive, so $252-5t > 0 \to t < 50.5 \to t \leq 50$. Then $20 \leq t \leq 50$ and there are $\boxed{31}$ values of t generating values of k and m where $k > m$. These range from $(k,m) = (160,152)$ to (400, 2).

4. The best way to do this is to graph $y=x^2-3$ and $y=2[x]$ and see where they intersect. If $x=-1$ then $2[x]=-2$ and $(-1)^2-3=-2$, so -1 is a solution. Similarly, if $x=3$ both sides equal 6 so 3 is a solution. There seems to be a solution between 2 and 3, however. We note that the values of x^2-3 lie between 1 and 6 on that interval. The value of $2[x]$ is just 4. Since $\sqrt{7}^2-3=4$, then $\sqrt{7}$ is a solution. The solutions are $\boxed{-1,\ \sqrt{7},\text{ and }3}$.



5. Consider the graph of $y=\dfrac{1}{|x-0.3|}+\dfrac{1}{|x-0.8|}$. On $[0,.3)$, since we have the sum of two positive numbers and a vertical asymptote at $x=.3$, the graph will increase without bound from left to right. On $[0,.3)$ the minimum value will, therefore, occur at $x=0$ and that value is $\dfrac{1}{\frac{3}{10}}+\dfrac{1}{\frac{4}{5}}=\dfrac{10}{3}+\dfrac{5}{4}=\dfrac{55}{12}$ which is less than 5. Similarly, on $(.8,1]$ the minimum will be $y(1)=\dfrac{1}{\frac{7}{10}}+\dfrac{1}{\frac{2}{10}}=\dfrac{45}{7}$ which is greater than 6. On $(.3,.8)$ the sum of the denominators is constant: $|x-.3|+|x-.8|=x-.3+.8-x=.5$. Note that $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{a+b}{ab}$ and from the AM-GM inequality, $\dfrac{a+b}{2}\geq\sqrt{ab}$, we know that the greatest value of the product ab occurs when $a=b$. Thus, on $(.3,.8)$, the sum $\dfrac{1}{|x-0.3|}+\dfrac{1}{|x-0.8|}=$ $\dfrac{.5}{(x-.3)(.8-x)}$ takes on its smallest value when the denominator is the largest and that occurs when $x-.3=.8-x\rightarrow x=.55$. We have $y(.55)=\dfrac{1}{.25}+\dfrac{1}{.25}=8$. Thus, the minimum value of the expression on $[0,1]$ is $\boxed{\dfrac{55}{12}}$.

6. If 2 is the least number chosen, the other two numbers can only differ by 1, giving 234, 245, . . . , 29(10). There are 7 such sets.

If 3 is the least number chosen, the other two numbers can differ by 1 or 2, giving 345, 346, 356, 357, . . , 389, 38(10), and a singleton 39(10). There are $2\cdot5+1=11$ such sets.

If 4 is the least number chosen, there other two can differ by 1, 2, or 3, giving 456, 457, 458, 467, 468, 469, 478, 479, 47(10), a double: 489, 48(10), and a single: 49(10). There are $3\cdot3+2+1=12$ such sets.

If 5 is the least number chosen, the others can differ by 1 through 4, giving 567, 568, 569, 56(10), then a triple: 578, 579, 57(10), a double: 589, 58(10), and a single: 59(10). There are $1\cdot4+3+2+1=10$ such sets.

If 6 is the least number chosen, the others can differ by 1 through 5, giving a triple: 678, 679, 67(10), a double: 689, 68(10), and a single: 69(10) for a total of 6.

If 7 is the least number chosen, we have a double: 789, 78(10), for 2 sets.

If 8 is the least number chosen, we have 89(10) for 1 set.

There are 7+11 +12 + 10 + 6 + 2 + 1 = $\boxed{49}$ sets.