MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2019

Round 1 **Arithmetic and Number Theory**

1. (________ , ________)

2. (________ , ________)

3. ____________________

1. $5np2$ is divisible by 6 and $6p2$ is divisible by 4. $5np2$ and $6p2$ are 4-digit and 3-digit base 10 numerals, respectively. Compute the number of distinct ordered pairs (n, p).

2. If $81_b = 63_c$ for positive integer bases b and c, compute the ordered pair (b, c), where $b > 10$, AND $b+c$ IS AS SMALL AS POSSIBLE.

3. A 3-by-2-by-2 box is disassembled into 1 by 1 by 1 cubes some of which are missing faces that were shared with other cubes. Compute the number of faces shared by 1-by1-by-1 cubes.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

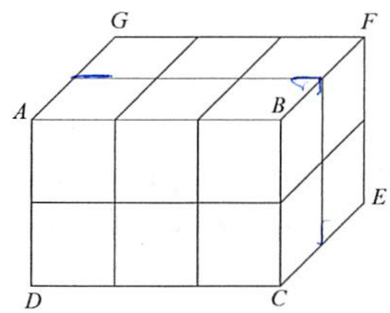
Solutions: State Meet 2019

Round 1: Arithmetic and Number Theory

1. For $6p2$ to be divisible by 4, p must be 1, 3, 5, 7, or 9. For $5np2$ to be divisible by 6, the digits must add to a multiple of 3, implying that $7+n+p=3k \Rightarrow (n+p)=2,5,8,11,14,17$
Thus, the possible ordered pairs are:
(1,1), (4,1), (7,1), (2,3), (5,3), (8,3), (0,5), (3,5), (6,5), (9,5), (1,7), (4,7), (7,7)
(2,9), (5,9), (8,9) $\rightarrow$ Answer: $3 \cdot 5 + 1 =$ **16**

2. $81_b = 63_c \rightarrow 8b+1 = 6c+3 \rightarrow 4b = 3c+1$. For b to be an integer, c must equal a number of the form $4k+1$. The first such number greater than 10 is 13. If $c=13$, then $4b=40$, so $b=10$, which is rejected, because of the requirement that $b>10$. If $c=17$, then $4b=52$, so $b=13$.
Check: $81_{13} = 63_{17} \rightarrow 8 \cdot 13 + 1 = 6 \cdot 17 + 3 = 105$ Answer: **(13, 17)**

3. Consider the 3 by 2 faces that are vertical and facing forward, one of which is *ABCD*. Moving from front to back there are 3 such faces making a total of $6 \cdot 3 = 18$ such 1 by 1 faces. Consider the 2 by 2 faces that are vertical and running from front to back, one of which is *BFEC*. There are 4 of those making a total of $4 \cdot 4 = 16$ such 1 by 1 faces. Finally, consider the 3 by 2 faces that are horizontal, one of which is *BFGA*. There are 3 of those making a total of $6 \cdot 3 = 18$ such 1 by 1 faces.
Total: 18 + 16 + 18 = **52**.



ADDITIONAL ANS. TO 2. AS ORIGINALLY WRITTEN

(13, 17)
(16, 21)
(19, 25)
$(3n+10, 4n+13)$

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2019

Round 2 **Algebra 1**

1. ______________________

2. ______________________

3. ______________________

1. Compute $\dfrac{\sqrt{\frac{3}{5}} \cdot 4^{-\frac{3}{2}}}{\left(\frac{1}{2\sqrt{5}}\right)^{3}}$.

2. Let b and c be positive integers.
If $x^2 + \dfrac{x}{b} + \dfrac{1}{c} = 0$ has two distinct real solutions, compute the <u>least</u> possible value of c.

3. Let a and b be members of $\{1, 2, 3, 4, 5\}$ with $a \neq b$.
Compute the <u>smallest</u> real value of x satisfying $5ax - 2bx = 4 - ax$.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

Solutions: State Meet 2019

Round 2 **Algebra I**

1. Simplifying the numerator and multiplying out the denominator, we have $\dfrac{\frac{\sqrt{15}}{5}\cdot\frac{1}{8}}{\frac{1}{40\sqrt{5}}} = \sqrt{75} = \boxed{5\sqrt{3}}$.

2. $x^2 + \dfrac{x}{b} + \dfrac{1}{c} = 0 \rightarrow bcx^2 + cx + b = 0$. The quadratic formula gives $x = \dfrac{-c \pm \sqrt{c^2 - 4b^2c}}{2bc}$. For the solutions to be real and distinct, $c^2 - 4b^2c > 0$ and, since $c > 0$ we can divide by c to obtain $c - 4b^2 > 0$. Since the least value of b is 1, $c > 4b^2$ gives $\boxed{5}$ as the least value of c.

3. $5ax - 2bx = 4 - ax \rightarrow 6ax - 2bx = 4 \rightarrow x = \dfrac{2}{3a-b}$. We might think that the smallest value for x would occur for a as large as possible and b as small as possible, i.e., $a = 5$ and $b = 1$, giving $x = \dfrac{2}{15-1} = \dfrac{1}{7}$, but we must consider the possibility of negative values. In that case, we'd like the denominator to be negative with an absolute value as small as possible. We obtain that with $a = 1$ and $b = 4$, giving $x = \dfrac{2}{3\cdot 1 - 4} = \boxed{-2}$.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2019

Round 3 – Geometry

1. ____________________

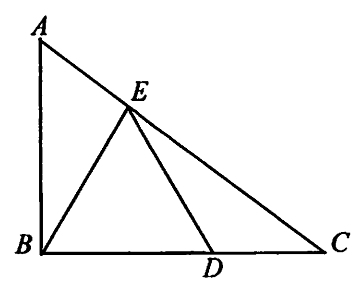
2. ____________________

3. ____________________

1. An equilateral triangle and a square share a common side. On the opposite side of the square from the triangle a regular hexagon shares a side of the square. The perimeter of the region bounded by the three polygons is 72 cm. A line segment is composed of an altitude of the triangle extended through the square to the opposite side of the hexagon. Compute the number of centimeters in the length of this line segment.

2. $ABCD$ is a rhombus of side 8 in; $\overline{AC}$ is the longer diagonal. The difference in area between a circle whose diameter is $\overline{AC}$ and one whose diameter is $\overline{BD}$ is 36π in. Compute the number of inches in the length of $\overline{BD}$.

3. Equilateral triangle BDE is inscribed in right triangle ABC where $m\angle B = 90°$, $AB = 6$, and $BC = 8$. The side of the equilateral triangle can be written in simplest form as $\frac{a\sqrt{b}+c}{13}$. Compute $a+b+c$.

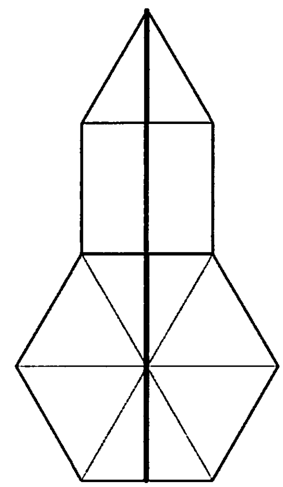


MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

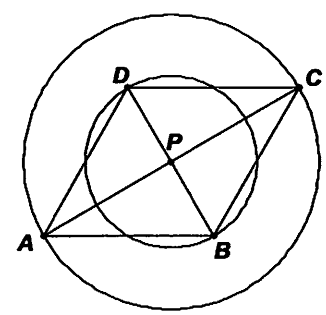
Solutions: State Meet 2019

Round 3 **<u>Geometry</u>**

1. The perimeter consists of 9 congruent sides, each of which must have a length of 8 cm. The hexagon can be divided into 6 congruent equilateral triangles. The required segment consists of an altitude of the equilateral triangle, $4\sqrt{3}$ cm, two altitudes from the triangles of the hexagon, $8\sqrt{3}$ cm, and the segment through the middle of the square, 8 cm.
   Answer $\boxed{\mathbf{8+12\sqrt{3}}}$



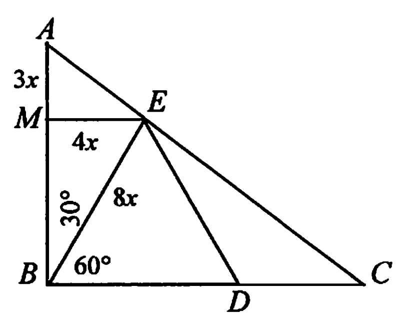
2. We have $PC^2 + DP^2 = 64$ and
   $\pi \cdot PC^2 - \pi \cdot DP^2 = 36\pi \to PC^2 - DP^2 = 36$.
   Adding gives $2 \cdot PC^2 = 100 \to PC^2 = 50$.
   Then $DP^2 = 14$ and that makes $\boxed{BD = 2\sqrt{14}}$.



3. Draw $\overline{ME}$ parallel to $\overline{BC}$, making $\Delta AME \sim \Delta ABC$.
   Let $AM = 3x$ and $ME = 4x$. Since $m\angle EBD = 60$, then ΔBME is a 30-60-90 right triangle, making $MB = \left(4\sqrt{3}\right)x$. Then

   $AB = 3x + 4x\sqrt{3} = 6 \to x = \dfrac{6}{4\sqrt{3}+3} \cdot \dfrac{4\sqrt{3}-3}{4\sqrt{3}-3} = \dfrac{8\sqrt{3}-6}{13}$.

   Since $BE = 8x$, we have $BE = \dfrac{64\sqrt{3}-48}{13}$. Then
   $a + b + c = 64 + 3 - 48 = \boxed{19}$.



MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2019

Round 4 – Algebra 2

1. ______________________

2. ______________________

3. ______________________

1. If $9^{(x+6)/2} = \left(\frac{1}{27}\right)^{8-2x}$, compute x.

2. If the value of the determinant is 0 and $x \neq y$, compute $x+y$.

$$\begin{vmatrix} 1 & 5 & 10 \\ 4 & x & y \\ 4 & y & x \end{vmatrix}$$

3. Compute the largest integer value of k such that $\frac{1}{\log_9 2} + \frac{1}{\log_{25} 4} > k$?

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

Solutions: State Meet 2019

Round 4: <u>Algebra II</u>

1. Changing 9 to 3^2 and $\frac{1}{27}$ to 3^{-3}, then equating exponents, we have $x + 6 = 6x - 24 \rightarrow \boxed{x = 6}$.

2. $$\begin{vmatrix} 1 & 5 & 10 \\ 4 & x & y \\ 4 & y & x \end{vmatrix} = 1\left(x^2 - y^2\right) - 5(4x - 4y) + 10(4y - 4x) = \left(x^2 - y^2\right) - 60(x - y).$$
Thus, $(x-y)(x+y) - 60(x-y) = 0$ gives $(x-y)(x+y-60) = 0$. So $\boxed{x + y = \mathbf{60}}$.

3. $$\frac{1}{\log_9 2} + \frac{1}{\log_{25} 4} > k \rightarrow \log_2 9 + \log_4 25 > k \rightarrow \log_2 9 + \log_2 5 > k \rightarrow \log_2 45 > k.$$
Thus, from $45 > 2^k$, we conclude that the <u>largest</u> integer value of k is $\boxed{5}$.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2019

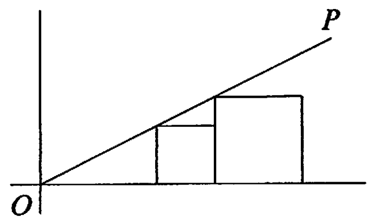
Round 5 – Analytic Geometry

1. ____________________

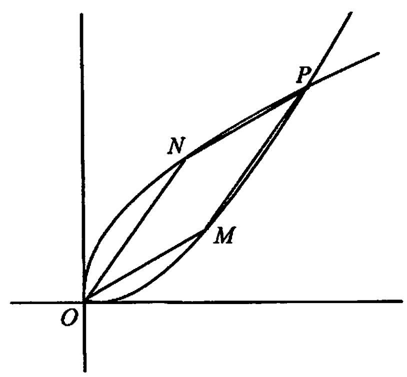
2. ____________________

3. ____________________

1. $\overline{OP}$ contains the origin and passes through the vertices of squares whose areas are 16 and 25 as shown in the diagram. Compute the slope of $\overline{OP}$.



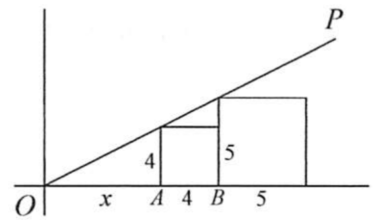
2. Shown are the graphs of $y = x^2$ and $y = \sqrt{x}$. O is the origin and P is the intersection point of the two graphs. M and N lie on $y = x^2$ and $y = \sqrt{x}$, respectively. The coordinates of M are (a, b) and $OMPN$ is a rhombus. Compute $a + b$.



3. Point $P(a,b)$ is equidistant from $y = x$, $y = -x$, and the point (6, 2). Compute the largest possible value of $a + b$.

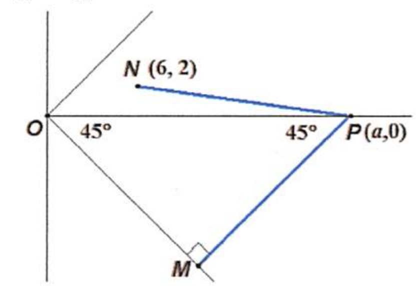
Solutions: State Meet 2019

Round 5: Analytic Geometry

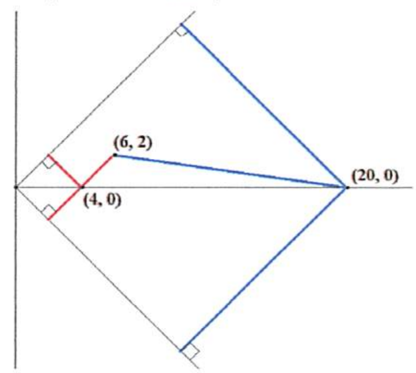


1. Let $OA = x$ making $OB = x + 4$. Then $\frac{5}{x+4} = \frac{4}{x} \to 5x = 4x + 16 \to x = 16$. The slope of $\overline{OP}$ is $\boxed{\frac{1}{4}}$.

2. Since $P = (1, 1)$ and $\overline{NM} \perp \overline{OP}$, then N is the reflection of M across $y = x$, making $N = (b, a)$. Thus, the slope of $PM = \frac{1-b}{1-a}$ and the slope of $NO = \frac{a}{b}$. Since $OMPN$ is a rhombus, the slopes are equal. Setting $\frac{1-b}{1-a} = \frac{a}{b}$ gives $\left(a^2 - b^2\right) - (a - b) = 0$. Then $(a-b)(a+b) - (a-b) = 0 \to (a-b)(a+b-1) = 0$, so $\boxed{a + b = \mathbf{1}}$.
   Alternate solution 1: Since $P = (1, 1)$ and $\overline{NM} \perp \overline{OP}$, then N is the reflection of M across $y = x$, making $N = (b, a)$. Since $OMPN$ is a rhombus, $MO = PM$, giving $\sqrt{a^2 + b^2} = \sqrt{(1-a)^2 + (1-b)^2}$.
   This simplifies to $2a + 2b = 2$ so $a + b = \boxed{1}$. From $M(a, b)$ lying on $y = x^2$ we know that $a^2 = b$ so we can find a and b by substitution: $a + a^2 = 1$ gives $a = \frac{-1+\sqrt{5}}{2}$ and $b = \frac{3-\sqrt{5}}{2}$.
   Note that a is the reciprocal of the Golden Ratio and b is the reciprocal squared.
   Alternate Solution 2: $M = (a, b) = (a, a^2)$. $P = (1,1)$. $PM = MO$. By the distance formula: $(1 - a^2)^2 + (1 - a)^2 = a^2 + a^4$. $\to 2 - 2a^2 - 2a = 0 \to a^2 + a - 1 = 0$.
   Since $a^2 = b$, $a + b = \boxed{1}$.



3. Points equidistant from $y = x$ and $y = -x$ lie on the x- and y-axes. Since (6, 2) lies in the first quadrant, the points we are looking for lie on the *positive x-axis*, so $P = (a, 0)$ and $b = 0$.
   Since $OP = a$ and ΔOPM is a 45-45-90 right triangle, then $PM = \frac{a}{\sqrt{2}}$. From $PM = PN$ we obtain $\frac{a}{\sqrt{2}} = \sqrt{(a-6)^2 + 2^2}$ which simplifies to $a^2 - 24a + 80 = 0 \to (a-4)(a-20) = 0 \to a = 4$ or 20. The largest value of $a + b = 20 + 0 = \boxed{20}$.



MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2019

Round 6 – Trig and Complex Numbers

1. ____________________

2. ____________________

3. ____________________

1. For $0° < \theta < 90°$, $\tan\theta = 2a$ and $\cos\theta = \dfrac{1}{a\sqrt{6}}$. Compute a.

2. Solve for x over $[0, 2\pi)$:

$$\frac{1}{1-\dfrac{1}{1-\dfrac{1}{1-\csc^2 x}}} = 4$$

3. The complex number $\sqrt[3]{\sqrt{\frac{1}{2}+\frac{\sqrt{3}}{2}i}}$ can be written in polar form as $r(\cos\theta + i\sin\theta)$.
If $r < 0$, compute the smallest possible positive value of θ in degrees.
Note: The outside radical is a cube root.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

Solutions: State Meet 2019

Round 6: Trigonometry and Complex Numbers

1. From $1+\tan^2\theta=\sec^2\theta$ we have $1+4a^2=\sec^2\theta$, so $\cos\theta=\dfrac{1}{\sqrt{1+4a^2}}$. Then:

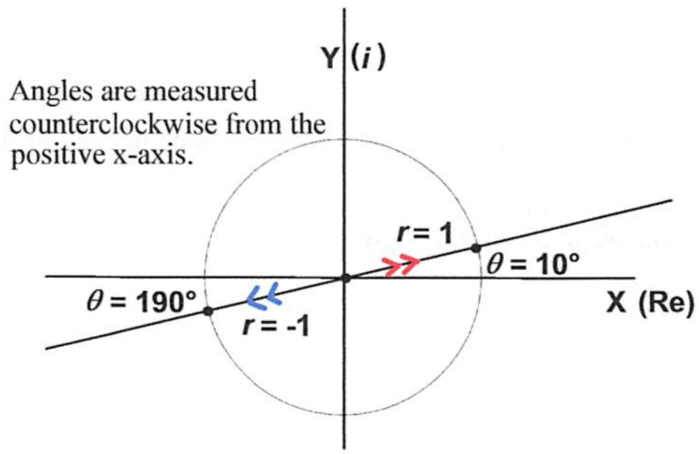
$$\frac{1}{a\sqrt{6}}=\frac{1}{\sqrt{1+4a^2}}\to 6a^2=1+4a^2\to a^2=\frac{1}{2}.$$ Since $\cos\theta>0$, $\boxed{a=\dfrac{\sqrt{2}}{2}}$.

2. $$\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\csc^2 x}}}=\cfrac{1}{1-\cfrac{1}{1+\cfrac{1}{\cot^2 x}}}=\cfrac{1}{1-\cfrac{1}{\sec^2 x}}=\frac{1}{\sin^2 x}=\csc^2 x=4.$$

Then: $\sin^2 x=\dfrac{1}{4}\to\sin x=\pm\dfrac{1}{2}\to x=\boxed{\dfrac{\pi}{6},\dfrac{5\pi}{6},\dfrac{7\pi}{6},\dfrac{11\pi}{6}}$.

3. Since $\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$ makes a 60° angle with the positive x-axis, its square root makes a 30° angle and the cube root of that makes a 10° angle. Converting to $r cis\theta$ form, the modulus of $\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$ is

$r=\sqrt{\left(\dfrac{1}{2}\right)^2+\left(\dfrac{\sqrt{3}}{2}\right)^2}=1$ and $\sqrt[3]{\sqrt{\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i}}=1(\cos 10+i\sin 10)$. When $r=-1$, $\theta=\boxed{190}$.



MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2019

Team Round

Place answers on Team Round Answer Sheet.

1. Let a and b be positive integers. Compute <u>all</u> values of b such that the points $M(a,1)$, $N(2,3)$, and $P(6,b)$ are collinear and distinct.

2. If $f(1)=1$ and $f(n)=f(n-1)+4n$ for $n\geq 2$, compute $f(100)$.

3. Let $f(x)=\log_4\left(\frac{x+3}{x-7}\right)$. Compute the number of integers that <u>cannot</u> be solutions to $f(x)<1$.

4. Let $a_1=1$, $a_2=3$, and $a_n=2a_{n-1}+a_{n-2}$ for $n>2$.
Compute the <u>smallest</u> possible solution to the quadratic equation $2a_{n-1}x^2+a_nx+a_{n-2}=0$.

5. The length of each leg of an isosceles trapezoid is the geometric mean between the shorter base and the longer base. What values can the common ratio take on?

6. Let N be the set of subsets of $S=\{1,2,3,\ldots,19,20\}$. A subset is chosen at random from N. Compute the probability that this subset contains <u>exactly</u> 2 primes.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

Solutions: State Meet 2019

Team Round

1. Since the slope of $\overline{NM}$ equals the slope of $\overline{PN}$ then $\frac{3-1}{2-a}=\frac{b-3}{6-2} \rightarrow 8=2b-6-ab+3a$. This simplifies to $b=\frac{14-3a}{2-a}$. Let $2-a=t$. Substitution gives $b=\frac{14-3(2-t)}{t}=3+\frac{8}{t}$. Since t must be a factor of 8 if b is an integer, we have the following table:

t	1	2	4	8	–1	–2	–4	–8
b	11	7	5	4	–5	–1	1	2
a	1	0	–2	–6	3	4	6	10

The solutions which give positive values for a and b are $a=1, b=11$ or $a=6, b=1$ or $a=10, b=2$. However, if $a=6$ and $b=1$, then $M = P = (6,1) = P = (6,1)$. Since the points must be distinct, we reject that solution. Thus, $b = \boxed{2 \text{ or } 11}$.

Alternate Solution:
From $b=\frac{3a-14}{a-2}$, by long division, $b=3-\frac{8}{a-2}$.
Now, since a and b are both positive, a = 1, 3, 4, 6, and 10.
Eliminate 3 and 4, since b would be negative.
Eliminate 6, since the points would <u>not</u> be distinct.
Thus, $b = \boxed{2 \text{ or } 11}$.

2. Note that the sequence telescopes:

$$\begin{aligned}
f(n) &= f(n-1)+4n \\
f(n-1) &= f(n-2)+4(n-1) \\
f(n-2) &= f(n-3)+4(n-2) \\
f(n-3) &= f(n-4)+4(n-3) \\
&\vdots \\
f(4) &= f(3)-4\cdot 4 \\
f(3) &= f(2)-4\cdot 3 \\
f(2) &= f(1)-4\cdot 2
\end{aligned}$$

Adding vertically and canceling all terms that appear on both sides of the equals sign gives an explicit formula for $f(n)$, namely,

$$f(n)=f(1)+4(n+(n-1)+(n-2)+\cdots+4+3+2) = 1+4\left(\frac{n(n+1)}{2}-1\right) = 2n^2+2n-3.$$

Then: $f(100)=2\cdot 100^2+200-3=\boxed{20{,}197}$.

Team Round - continued

3. Given: $\log_4\left(\frac{x+3}{x-7}\right)<1 \Leftrightarrow 0<\frac{x+3}{x-7}<4$

First, consider those numbers that *can be* solutions. Since the argument must be positive, either $(x+3>0 \text{ and } x-7>0)$ or $(x+3<0 \text{ and } x-7<0)$.

Case 1: $x+3>0$ and $x-7>0 \Leftrightarrow x-7>0$: Multiplying through by $x-7$ (which is a positive quantity), we have $0<x+3<4x-28 \rightarrow x>-3$ and $x>10\frac{1}{3} \rightarrow x>10\frac{1}{3}$.

Case 2: $x+3<0$ and $x-7<0 \Leftrightarrow x+3<0$: Multiplying through by $x+3$ (which is a negative quantity), we have $0>x+3>4x-28 \rightarrow x<-3$ and $x<10\frac{1}{3} \rightarrow x<-3$.

Thus, since solutions lie to the right of $10\frac{1}{3}$, or to the left of -3, there are *no* solutions for $-3 \le x \le 10$ and there are $10-(-3)+1=\boxed{14}$ integers that cannot be solutions to $f(x)<1$.

Alternate Solution:

To insure $f(x)=\log_4\left(\frac{x+3}{x-7}\right)<1$, we require that $\frac{x+3}{x-7}<4$.

To insure a real-valued function so that $f(x)<1$ makes sense, we require that $\frac{x+3}{x-7}>0$.

The first condition:

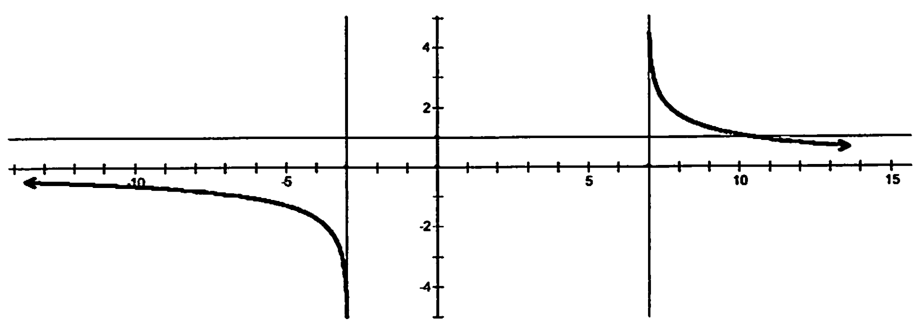
$$\frac{x+3}{x-7}<4 \Leftrightarrow \frac{x+3}{x-7}-4<0 \Leftrightarrow \frac{x+3-4x+28}{x-7}<0 \Leftrightarrow \frac{-3x+31}{x-7}<0 \Leftrightarrow \frac{3x-31}{x-7}>0$$

The critical values are $x=7, \frac{31}{3}$ and the inequality is valid for $\boxed{x<7 \text{ or } x>\frac{31}{3}}$.

The second condition requires numerator and denominator to be both positive or both negative. This occurs when $\boxed{x<-3 \text{ or } x>7}$. Taking the intersection, we have $x<-3$ or $x>\frac{31}{3}$.

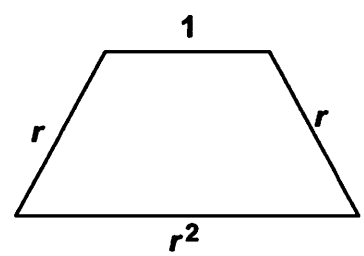
Thus, the integers that cannot be solutions are $\{-3,-2,-1,0,1,\ldots,10\}$, a total of **<u>14</u>** integers.

Here's the graph:



4. Since $a_n = 2a_{n-1} + a_{n-2}$, the equation becomes $2a_{n-1}x^2 + (2a_{n-1} + a_{n-2})x + a_{n-2} = 0$. This factors as $(2a_{n-1}x + a_{n-2})(x+1) = 0$. The solutions are $x_1 = -\dfrac{a_{n-2}}{2a_{n-1}}$ or $x_2 = -1$. Since $2a_{n-1} > a_{n-2}$, then $x_1 = -\dfrac{a_{n-2}}{2a_{n-1}} > -1$. Thus, the least possible solution is $\boxed{-1}$.

   Alternate Solution
   Applying the recursive definition, the first few values of $f(x)$ are 1, 3, 7, 17, 41, 99,... .
   This is, clearly, an increasing function.
   Substituting for a_n in the given quadratic equation, we have $2a_{n-1}x^2 + (2a_{n-1} + a_{n-2})x + a_{n-2} = 0$.
   Expanding and regrouping, we have $(2a_{n-1}x^2 + 2a_{n-1}x) + (a_{n-2}x + a_{n-2}) = 0$.
   Factoring out the common factor in each pair,
   $2a_{n-1}x(x+1) + a_{n-2}(x+1) = 0 \Leftrightarrow (x+1)(2a_{n-1} + a_{n-2}) = 0$.
   Thus, $x = -1$ or $x = -\dfrac{a_{n-2}}{2a_{n-1}}$.
   Since $f(x)$ defines an increasing function, $a_{n-2} < a_{n-1}$.
   $\Rightarrow a_{n-2} < 2a_{n-1} \Leftrightarrow \dfrac{a_{n-2}}{2a_{n-1}} < 1 \Leftrightarrow -\dfrac{a_{n-2}}{2a_{n-1}} > -1$.
   Thus, the <u>smallest</u> possible solution is $\boxed{-1}$.



5. Without loss of generality, we can set the lengths of the sides as $1, r, r$, and r^2, where 1 denotes the shorter base, r denotes the length of the legs, and r^2 denotes the length of the longer base. For there to be a trapezoid, we must have $r + 1 + r > r^2 \to 0 > r^2 - 2r - 1$.
   The graph of $y = r^2 - 2r - 1 = (r-1)^2 - 2$ is a concave up parabola, and the solution we seek lies between the zeros. Solving $r^2 - 2r - 1 = 0$ gives $r = 1 \pm \sqrt{2}$. So $1 - \sqrt{2} < r < 1 + \sqrt{2}$. Because lengths are involved, r can't be less than or equal to 0, and because r^2 is the longest side, $r^2 > 1$.
   Thus, the possible r-values must satisfy $\boxed{1 < r < \sqrt{2} + 1}$.

6. There are 8 primes in S and 12 non-primes. N has 2^{20} subsets. Two primes can be selected in $\binom{8}{2} = 56$ ways, and there are 2^{12} sets of non-primes. The probability that a subset chosen from N will have exactly two primes is, therefore, $\dfrac{56 \cdot 2^{12}}{2^{20}} = \boxed{\dfrac{7}{32}}$.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2019 – ANSWER SHEET

Round 1

1. 16
2. (13, 17)
3. 52

Round 2

1. $5\sqrt{3}$
2. 5
3. -2

Round 3

1. $8 + 12\sqrt{3}$
2. $2\sqrt{14}$
3. 19

Round 4

1. 6
2. 60
3. 5

Round 5

1. $\frac{1}{4}$
2. 1
3. 20

Round 6

1. $\frac{\sqrt{2}}{2}$
2. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
3. 190

Team

1. 2 or 11
2. 20,197
3. 14
4. -1
5. $1 < r < 1 + \sqrt{2}$
6. $\frac{7}{32}$